

NOTES AND ERRATA, VOLUMES 10, 11.

VOLUME 10.

Page 315. W. B. FITE: *Irreducible homogeneous linear groups in an arbitrary domain.*

The results of this article are embodied in the theorem, *A necessary and sufficient condition that any group of finite order be simply isomorphic with an irreducible group in any domain is that its central be cyclic.* But that this condition is not sufficient is shown by the following example to which Professor BURNSIDE has kindly called my attention, namely the group defined by the operations A , P , Q , and R with the following relations:

$P^7 = 1$, $Q^7 = 1$, $R^n = 1$ (n any positive integer), $A^3 = 1$, $A^{-1}PA = P^2$, $A^{-1}QA = Q^2$, $A^{-1}RA = R$, and with P , Q , and R commutative among themselves.

The argument rests upon the supposed fact that $r_1 > 1$, and I can now see no reason why it might not equal unity.

I have been unable to determine what is both a sufficient and a necessary condition that a group of finite order may be simply isomorphic with an irreducible group. It is easy however to give sufficient conditions that are not necessary ones; for example, in order that a group of finite order be simply isomorphic with an irreducible group it is *sufficient* that its direct product with some group of finite order be simply isomorphic with an irreducible group.

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Page 489. G. E. WAHLIN: *On the base of a relative number field, with an application to the composition of fields.*

The assertion that $O_{\tau\tau}^{(1)}$ is prime to I_τ is not true for every ϑ . I wish therefore to replace lines 7-19 of p. 489 by the following.

Since $\omega_{\tau\tau}^{(1)} = \bar{\omega}_\tau / D_k(\vartheta)$ is an integer in K , $\bar{\omega}_\tau$ is divisible by $D_k(\vartheta)$, and consequently also by O_τ . That is,

$$\bar{\omega}_\tau = O_{1\tau}^{(1)} + O_{2\tau}^{(1)}\vartheta + \dots + O_{\tau\tau}^{(1)}\vartheta^{\tau-1} \equiv 0(O_\tau).$$

From the fact,* that if J and J_1 are two ideals in a number field, then there exists an ideal J' such that JJ' is a principal ideal and J' is prime to J_1 , we know that there exists in k an ideal J_τ , prime to $[D_k(\vartheta)]$ and such that

* DIRICHLET: *Vorlesungen über Zahlentheorie*, vierte Auflage, p. 559, Theorem XI.

$O_\tau \cdot J_\tau = (\alpha)$, a principal ideal. In the same way there exists an ideal J'_τ , prime to $[D_k(\vartheta)]$, such that $J_\tau \cdot J'_\tau = (\beta)$, a principal ideal, and since J_τ and J'_τ are both prime to $D_k(\vartheta)$, β is prime to $D_k(\vartheta)$.

Since $(\beta) = J_\tau \cdot J'_\tau$ and $(\alpha) = O_\tau \cdot J_\tau$ and since $\bar{\omega}_\tau$ is divisible by O_τ , it follows that $\beta \bar{\omega}_\tau$ is divisible by (α) , and must therefore be a multiple of α , and

$$\frac{\beta \bar{\omega}_\tau}{\alpha} = \frac{\beta O_{1\tau}^{(1)} + \beta O_{2\tau}^{(1)} \vartheta + \dots + \beta O_{\tau\tau}^{(1)} \vartheta^{\tau-1}}{\alpha}$$

is an integer in K .

But since $\bar{\omega}_\tau$ and $O_{\tau\tau}^{(1)}$ are both divisible by O_τ it follows that

$$X = O_{1\tau}^{(1)} + O_{2\tau}^{(1)} \vartheta + \dots + O_{\tau-1,\tau}^{(1)} \vartheta^{\tau-2} = \bar{\omega}_\tau - O_{\tau\tau}^{(1)} \vartheta^{\tau-1}$$

must be divisible by O_τ and therefore, in the same way as above, we see that $\beta X/\alpha$ is an integer in K , and $R = \beta O_{\tau\tau}^{(1)}/\alpha$ an integer in k . We can now write

$$\frac{\beta \bar{\omega}_\tau}{\alpha} = \frac{\beta X + \beta O_{\tau\tau}^{(1)} \vartheta^{\tau-1}}{\alpha} = \frac{\beta X}{\alpha} + R \vartheta^{\tau-1}$$

and since O_τ is the highest common factor of $O_{\tau\tau}^{(1)}$ and $D_k(\vartheta)$, the ideals $O_{\tau\tau}^{(1)}/O_\tau$ and $I_\tau = D_k(\vartheta)/O_\tau$ are relatively prime, and since β is prime to I_τ , it follows that R is prime to I_τ and there exists in k an integer R' such that

$$R'R \equiv 1 (I_\tau).$$

We have therefore

$$\frac{R'\beta \bar{\omega}_\tau}{\alpha} = \frac{R'\beta X}{\alpha} + R'R \vartheta^{\tau-1} \equiv \bar{\Omega}_\tau (I_\tau),$$

where

$$\bar{\Omega}_\tau = \frac{R'\beta O_{1\tau}^{(1)} + R'\beta O_{2\tau}^{(1)} \vartheta + \dots + R'\beta O_{\tau-1,\tau}^{(1)} \vartheta^{\tau-2}}{\alpha} + \vartheta^{\tau-1}.$$

Since $\bar{\omega}_\tau$ is divisible by $D_k(\vartheta)$, and α contains no other factor of $D_k(\vartheta)$ than O_τ , we see that the integer $\beta \bar{\omega}_\tau/\alpha$ must be divisible by I_τ and from the last congruence we have

$$\bar{\Omega}_\tau \equiv O(I_\tau).$$

While $\Omega_\tau = \bar{\Omega}_\tau/D_k(\vartheta)$ is not necessarily an integer in K , the products $\Omega_\tau \cdot O_{\tau\tau}^{(1)}$ and $\Omega_\tau \cdot D_k(\vartheta)$ are both integers in K . Moreover, since β is divisible by J_τ and since $D_k(\vartheta)$ and $O_{\tau\tau}^{(1)}$ are both divisible by O_τ , the coefficients $R'\beta O_{\tau\tau}^{(1)}$ when multiplied by $O_{\tau\tau}^{(1)}$ or $D_k(\vartheta)$ will be divisible by α and the integers $\Omega_\tau \cdot D_k(\vartheta)$ and $\Omega_\tau \cdot O_{\tau\tau}^{(1)}$ are therefore of the form $\omega_\tau^{(i)}$. The number Ω_τ thus found can therefore take the place of the Ω_τ in my original paper.